Chapter 26

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Capacitance And Dielectrics

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Lecture 04

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Capacitance With Dielectrics

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Capacitors with Dielectrics

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A dielectric is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance. *(rubber, glass, and waxed paper)*

The capacitance increases by a factor (κ) when the dielectric **completely** fills the region between the plates.

$$C = \kappa C_\circ$$

- C_o : Capacitor without a dielectric
- C : Capacitor with a dielectric
- κ : is the dielectric constant of the material. (Dimensionless)

$$\kappa > 1$$

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Capacitors with Dielectrics

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Capacitance	• $C = \kappa C_\circ$
Electric permittivity	• $\epsilon = \kappa \epsilon_\circ$
Coulomb's constant	• $k_{dielectric} = \frac{1}{4\pi\epsilon} = \frac{1}{4\pi\kappa\epsilon_\circ} = \frac{9\times10^9}{\kappa}\ N.m^2/C$
Gauss's Law	• $\Phi_{dielectric} = \frac{q_{in}}{\epsilon} = \frac{q_{in}}{\kappa\epsilon_\circ}$

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Capacitors with Dielectrics, (FIXED CHARGE)

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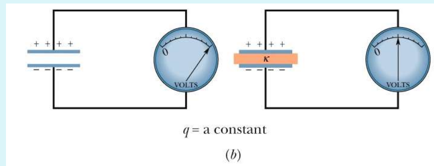
- If the capacitor is disconnected from the battery, the capacitor is an isolated system and the charge remains the same, while the voltage will decrease.



q = a constant

(b)

Capacitors with Dielectrics, (FIXED CHARGE)

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Capacitance: increases	• $C = \kappa C_\circ$
Charge: remains constant	• $Q = Q_\circ$
Potential difference: decreases	• $\Delta V = \frac{\Delta V_\circ}{\kappa}$
Energy: decreases	• $U = \frac{U_\circ}{\kappa}$
Electric field: decreases	• $\vec{E} = \frac{\vec{E}_\circ}{\kappa}$

Capacitors with Dielectrics, (FIXED POTENTIAL)

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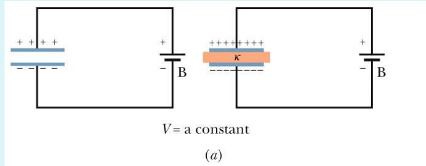
- If the capacitor remains connected to a battery, the voltage across the capacitor remains the same, while the charge will increase.



Capacitors with Dielectrics, (FIXED POTENTIAL)

8

Capacitance: increases	• $C = \kappa C_\circ$
Charge: increases	• $Q = \kappa Q_\circ$
Potential difference: remains constant	• $\Delta V = \Delta V_\circ$
Energy: increases	• $U = \kappa U_\circ$
Electric field: remains constant	• $\vec{E} = \vec{E}_\circ$

Dielectrics, cont

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- For a parallel-plate capacitor,

$$C = \frac{\kappa \epsilon_\circ A}{d}$$

- In theory, d could be made very small to create a very large capacitance.
- In practice, there is a limit to d.

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Dielectrics, final

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- Dielectrics provide the following advantages:
  - Increase in capacitance
  - Increase the maximum operating voltage
  - Possible mechanical support between the plates
    - This allows the plates to be close together without touching.
    - This decreases d and increases C.

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Dielectrics – An Atomic View

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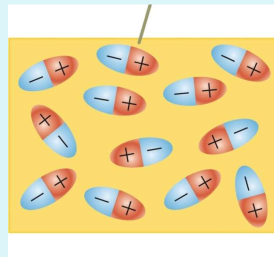
- The molecules that make up the dielectric are modeled as dipoles.
- The molecules are randomly oriented in the absence of an electric field.



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Dielectrics – An Atomic View, 2

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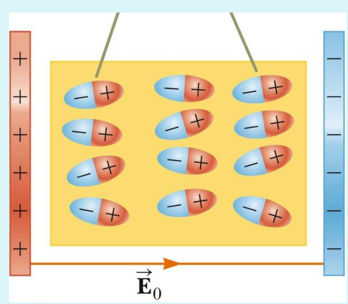
- An external electric field is applied.
- This produces a torque on the molecules.
- The molecules partially align with the electric field.
  - The degree of alignment depends on temperature and the magnitude of the field.
    - In general, the alignment increases with decreasing temperature and with increasing electric field.



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Dielectrics – An Atomic View, final

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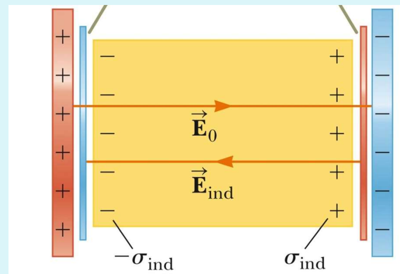
- An external field can polarize the dielectric.
- The charged edges of the dielectric act as a second pair of plates producing an induced electric field in the direction opposite the original electric field.

$$E = E_\circ - E_{induced}$$



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Induced Charge and Field

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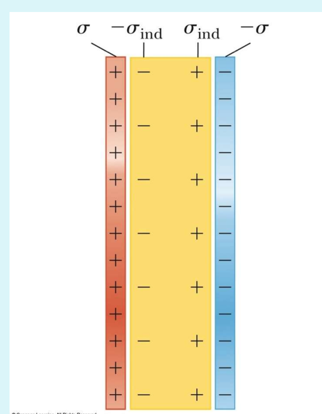
- The electric field due to the plates is directed to the right.
- The net effect on the dielectric is an induced surface charge that results in an induced electric field.

$$\sigma_{induced} = \left(\frac{\kappa - 1}{\kappa}\right)\sigma$$

- If the dielectric were replaced with a conductor, the net field between the plates would be zero.



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Some Dielectric Constants and Dielectric Strengths

Table 26.1 Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a]The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

Work and energy when a dielectric is inserted into a capacitor

Tuesday, 2 February, 2021 19:59

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A parallel-plate capacitor whose capacitance $C = 13.5\, pF$ is charged by a battery to a potential difference $V = 12.5\, V$ between its plates. The charging battery is now disconnected, and a porcelain slab $(\kappa = 6.50)$ is slipped between the plates.

- What is the potential energy of the capacitor before the slab is inserted?
- What is the potential energy of the capacitor–slab device after the slab is inserted?

Because we are given the initial potential $(V = 12.5V)$, find the initial stored energy by using:

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}(13.5 \times 10^{-12}F)(12.5V)^2$$

$$= 1.055 \times 10^{-9}J = 1055pJ \approx 1100pJ.$$

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change.

Thus, we must now use $(U = \frac{q^2}{2C})$ to write the final potential energy U_f, but now that the slab is within the capacitor, the capacitance is kC. We then have

$$U_f = \frac{q^2}{2kC} = \frac{U_i}{k} = \frac{1055pJ}{6.50}$$

$$= 162pJ \approx 160pJ.$$

When the slab is introduced, the potential energy decreases by a factor of k.

The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162)pJ = 893pJ.$$

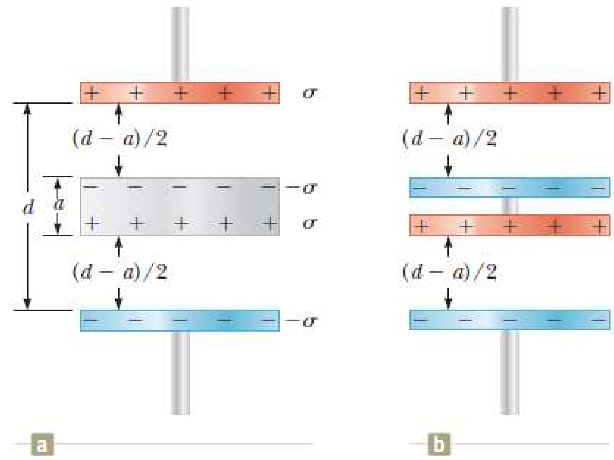
Effect of a Metallic Slab

Tuesday, 2 February, 2021 17:02

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A parallel-plate capacitor has a plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted midway between the plates.

- Find the capacitance of the device.
- Show that the capacitance of the original capacitor is unaffected by the insertion of the metallic slab if the slab is infinitesimally thin.



SOLUTION

Figure a shows the metallic slab between the plates of the capacitor. Any charge that appears on one plate of the capacitor must induce a charge of equal magnitude and opposite sign on the near side of the slab as shown in Figure a. Consequently, the net charge on the slab remains zero and the electric field inside the slab is zero.

we can model the edges of the slab as conducting planes and the bulk of the slab as a wire. As a result, the capacitor in Figure a is equivalent to two capacitors in series, each having a plate separation $(d-a)/2$ as shown in Figure b.

Find the equivalent capacitance in Figure b:

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{\frac{\epsilon_\circ A}{(d-a)/2}}+\frac{1}{\frac{\epsilon_\circ A}{(d-a)/2}}$$

$$C=\frac{\epsilon_\circ A}{d-a}$$

(B) In the result for part (A), let $a\to 0$:

$$C=\lim_{a\to 0}\left(\frac{\epsilon_\circ A}{d-a}\right)=\frac{\epsilon_\circ A}{d}$$

The result of part (B) is the original capacitance before the slab is inserted,

What if the metallic slab in part (A) is not midway between the plates? How would that affect the capacitance?

Let's imagine moving the slab in Figure a upward so that the distance between the upper edge of the slab and the upper plate is b. Then, the distance between the lower edge of the slab and the lower plate is $d-b-a$. As in part

(A), we find the total capacitance of the series combination:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\epsilon_\circ A/b} + \frac{1}{\epsilon_\circ A/(d-b-a)} = \frac{b}{\epsilon_\circ A} + \frac{d-b-a}{\epsilon_\circ A} = \frac{d-a}{\epsilon_\circ A}$$

$$\rightarrow C = \frac{\epsilon_\circ A}{d-a}$$

which is the same result as found in part (A). The capacitance is independent of the value of b, so it does not matter where the slab is located.

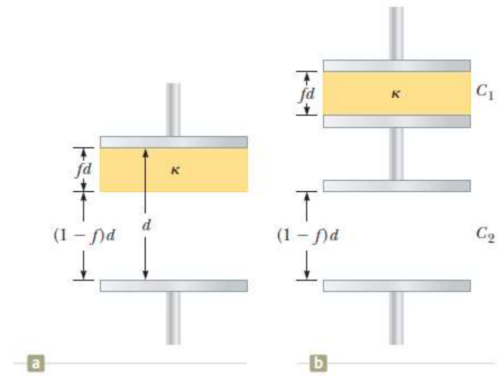
A Partially Filled Capacitor

Tuesday, 2 February, 2021 17:05

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A parallel-plate capacitor with a plate separation d has a capacitance C_o in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant κ and thickness fd is inserted between the plates, where f is a fraction between 0 and 1?



SOLUTION

Imagine sliding an infinitesimally thin metallic slab along the bottom face of the dielectric shown in Figure a.

One capacitor has a plate separation fd and is filled with a dielectric; the other has a plate separation $(1-f)d$ and has air between its plates.

Evaluate the two capacitances:

$$C_1 = \frac{k\epsilon_\circ A}{fd}$$

and

$$C_2 = \frac{\epsilon_\circ A}{(1-f)d}$$

Find the equivalent capacitance C for two capacitors combined in series:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{fd}{k\epsilon_\circ A} + \frac{(1-f)d}{\epsilon_\circ A}$$

$$\frac{1}{C} = \frac{fd}{k\epsilon_\circ A} + \frac{k(1-f)d}{k\epsilon_\circ A} = \frac{f+k(1-f)}{k}\frac{d}{\epsilon_\circ A}$$

Invert and substitute for the capacitance without the dielectric, $C_0 = \epsilon_\circ A/d$:

$$C = \frac{k}{f+k(1-f)}\frac{\epsilon_\circ A}{d} = \frac{k}{f+k(1-f)}C_0$$

If $f \to 0$, the dielectric should disappear. In this limit, $C \to C_\circ$, which is consistent with a capacitor with air between the plates.

If $f \to 1$, the dielectric fills the volume between the plates. In this limit, $C \to \kappa C_\circ$.

A capacitor with and without a dielectric

Friday, 29 January, 2021 21:02

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

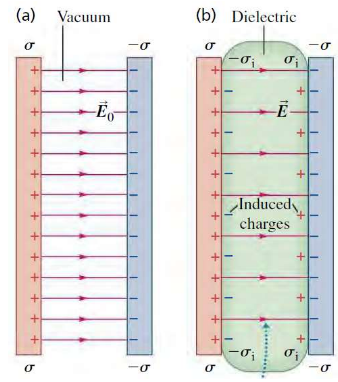
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Suppose the parallel plates shown in the figure each have an area of $2000\ cm^2$ and are $1\ cm$ apart. We connect the capacitor to a power supply, charge it to a potential difference $V_\circ = 3\ kV$, and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to $1\ kV$ while the charge on each capacitor plate remains constant. Find:



- the original capacitance $C_\circ$;
- the magnitude of charge Q on each plate;
- the capacitance C after the dielectric is inserted;
- the dielectric constant κ of the dielectric;
- the permittivity ε of the dielectric;
- the magnitude of the induced charge Q_i on each face of the dielectric;
- the original electric field $E_\circ$ between the plates; and
- the electric field E after the dielectric is inserted.

With vacuum between the plates, $\kappa = 1$:

$$C_0 = \frac{A\epsilon_\circ}{d} = \frac{2000 \times 10^{-4} \times 8.85 \times 10^{-12}}{1 \times 10^{-2}} = 1.77 \times 10^{-10} F$$

From the definition of capacitance:

$$Q_0 = C_0 V_0 = 1.77 \times 10^{-10} \times 3 \times 10^3 = 5 \cdot 31 \times 10^{-7} C$$

When the dielectric is inserted, Q is unchanged but the potential difference decreases to $V = 1\ kV$. Hence, the new capacitance is:

$$C = \frac{Q}{V} = \frac{Q_0}{V} = \frac{5 \cdot 31 \times 10^{-7}}{1 \times 10^3} = 5.31 \times 10^{-10} F$$

the dielectric constant is:

$$k = \frac{C}{C_0} = \frac{5 \cdot 31 \times 10^{-10}}{1 \cdot 77 \times 10^{-10}} = 3$$

the permittivity is:

$$\epsilon = k\epsilon_\circ = 3 \times 8 \cdot 85 \times 10^{-12} = 2 \cdot 66 \times 10^{-11}\ C^2/N \cdot m^2$$

Multiplying both sides of $\sigma_{\text{ind}} = \sigma \left(\frac{k-1}{k}\right)$ by the plate area A, gives the induced charge $Q_{ind} = \sigma_{ind} A$ in terms of the charge $Q = \sigma A$ on each plate:

ind

$$Q_{ind} = Q\left(\frac{k-1}{k}\right) = 5.31 \times 10^{-7} \times \left(\frac{3-1}{3}\right) = 3.54 \times 10^{-7} C$$

Since the electric field between the plates is uniform, its magnitude is the potential difference divided by the plate separation:

$$E_0 = \frac{V_0}{d} = \frac{3 \times 10^3}{1 \times 10^{-2}} = 3 \times 10^5\ V/m$$

After the dielectric is inserted,

$$E = \frac{V}{d} = \frac{1 \times 10^3}{1 \times 10^{-2}} = 1 \times 10^5\ V/m$$

Inserting the dielectric increased the capacitance by a factor of $k = 3$ and reduced the electric field between the plates by a factor of $1/k = 1/3$. It did so by developing induced charges on the faces of the dielectric of magnitude:

$$Q\left(1 - \frac{1}{3}\right) = 0.667Q$$